Forced Convectional Heat Transfer in Solid Oxide Fuel Cells:
An Analytical Treatment.

F.A. Coutelieris[1], A.K. Demin[2], S.L. Douvartzides[1] and P.E. Tsiakaras[1]*

[1]University of Thessalia, Department of Mechanical and Industrial Engineering

Pedion Areos, 383 34, Volos, Greece.

[2]Institute of High Temperature Electrochemistry, Ekaterinburg, 620219,

S. Kovalevskoy str. 20, Russia.

Abstract. The heat transfer within a fuel cell at steady state conditions is considered here for one dimensional geometry. Analytical solution for the heat transfer equation accompanied by the appropriate boundary conditions is obtained. The heat transfer coefficient is also estimated for the case of ideal heat exchange. It was found that the geometrical characteristics of the cell that are strongly related with its electrical ones (namely, the ohmic resistance and the current developed), are favourable parameters for the maximization of the heat transfer.

1. Introduction

One of the most promising applications in the area of electricity production are fuel cells because they convert chemical energy directly to electricity presenting almost neglible impact in the aggravation of environment. Solid Oxide Fuel Cells (SOFCs), that are solid ceramic cells, seem to be the best option among all the types of fuel cells in the case that the operational temperature is high although this type of application faces some crucial designing problems such as self-diffusion of materials used, high stress etc. [1,3]. The electrochemical reaction for this energy conversion takes place in the cathode of a typical SOFC, producing electricity accompanying by a significant amount of heat, the management of which is one of the most important problems in SOFC designing [4]. There have been many recent works regarding several different formulations for the heat transfer problems within fuel cells, but all of them used numerical treatment to approach an accurate solution [5-8].

This work presents an analytical model for the thermal transport phenomena occurred within a SOFC stack operating at high temperatures. By considering the complete conduction-convection heat transfer equation along the cell, analytical expressions for the spatial distribution of both the gas and the cell temperatures as well as for the thermal conductivity are obtained by using an averaging technique. As the modeled fuel cell is a real cylindrical design and the electrochemistry considered is quite complete, the basic barriers mentioned above are overcome and the use of these analytical formulas can be very helpful in the design and/or in the operating phase of such systems, as the produced results are sufficiently accurate for any practical application.

2. Theory

A typical fuel cell stack consists of an anode and a cathode compartments between which a catalyst (electrolyte) layer exists. The atmospheric air flows in the cathode gas channel while the anode is exposed to hydrogen. Both oxygen and hydrogen are assumed to be enough for having the maximum allowable utilization of hydrogen, i.e. the difference of partial pressure of hydrogen is zero. The overall fuel cell reaction is in most cases a formation of water from hydrogen and oxygen, producing electricity and heat as follows:

* Corresponding author. E-mail: tsiak@mie.uth.gr, Telephone: +30421074065, Fax: +30421074050.

$$H_2 + \frac{1}{2} \, O_2 \rightarrow H_2O + heat + electricity \tag{1}$$
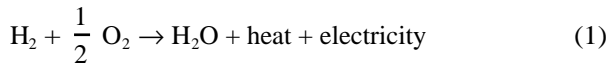
By considering an averaging procedure in the direction that is perpendicular to the gas flow, a consequent one-dimensional model results, having as basic variable the gas temperature, $T_g(x)$, averaged in a cross-section of the cathode as follows

$$T_g(x) = \frac{1}{L_y} \int_0^{L_y} T_g(x,y) \, dy \tag{2}$$

where $x = x/L_x$ is the dimensionless spatial coordination and L_x, L_y are arbitrarily defined characteristic lengths for the x- and y-direction, respectively. This averaging procedure should be applied separately in the solid and the gas phase, in order to avoid missing of information about the heat sources upon the solid-gas interface. Therefore, it is necessary for the heat transfer equation to be considered in both the two different shrinking phases.

The specific flux of the heat produced within the cell at any point x is constant

$$s = j \, (U_t + jr - E) \tag{3}$$

where $U_t = 1.29$ V denotes the thermo neutral voltage, and r denotes the specific resistance of the cell which is the summation of the ohmic and the anodic and cathodic polarization resistances. The electromotive force is constant at any point, x, of the domain [0,1] as has been previously presented by Tsiakaras and Demin [9].

As the conduction is the only available mechanism for the heat transport within solids, the heat transfer in the solid phase of the cell is described by the following differential equation:

$$-\alpha_s \frac{d^2 T_c(z)}{dz^2} = s = j \, (U_t + jr - E) \tag{4}$$

where α_s denotes the thermal diffusivity within the solid phase. After some simple mathematical manipulations, eq. (4) can be written as:

$$\frac{d^2 T_c(x)}{dx^2} = -\frac{j}{\alpha_s} \Big\{ U_t + jr - \frac{R\langle T_c(x)\rangle}{4F} \Big[\ln 0.209K^2 + 2\ln\Big(\frac{p_{H_2}(0)}{1-p_{H_2}(0)}\Big) \Big] \Big\} \tag{5}$$

having general solution of the form:

$$T_c(x) = -\frac{j}{2\alpha_s} \Big\{ U_t + jr_{ef} - \frac{R\langle T_c(x)\rangle}{4F} \Big[\ln 0.209K^2 + 2\ln\Big(\frac{p_{H_2}(0)}{1-p_{H_2}(0)}\Big) \Big] \Big\} x^2 + C_1 x + C_2 \tag{6}$$

By assuming quasi-isothermal operation for the cell, described by the boundary condition:

$$T_c(1) = T_c(0) \tag{7a}$$

and a commonly used Dirichlet boundary condition for the inlet temperature:

$$T_c(0) = constant \tag{7b}$$

the above-mentioned arbitrary constants are identified as follows:

$$C_1 = \frac{j}{\alpha_s} \Big\{ U_t + jr - \frac{R\langle T_c(x)\rangle}{4F} \Big[\ln 0.209K^2 + 2\ln\Big(\frac{p_{H_2}(0)}{1-p_{H_2}(0)}\Big) \Big] \Big\} \tag{8a}$$

$$C_2 = T_c(0) \tag{8b}$$

By considering the moving air as a compressible ideal gas and by assuming constant mass flow rate of the gas, $\overset{\bullet}{m} = \rho\Big(\frac{\pi f^2}{4}\Big) u$, through a circular surface of diameter d, the velocity $u(x)$ can be expressed as a function of gas temperature as follows:

$$u(x) = \frac{V_A^o \, \overset{\bullet}{m}}{\pi \frac{d^2}{4} MW_{air}} T_g(x) = 0.0036 \frac{\overset{\bullet}{m}}{d^2} T_g(x) \tag{9}$$

and, therefore, the differential equation describing the heat transfer in the gas phase can be written as:

$$0.0036 \frac{\overset{\bullet}{m}}{d^2} T_g(x) \frac{dT_g(x)}{dx} = \alpha_f \frac{d^2 T_g(x)}{dx^2} \tag{10}$$

where α_f denotes the thermal diffusivity within the solid phase. The solution of the above equation can be obtained by using a downgrading technique for non-linear differential equations [10] and is of the form:

$$T_g(x) = \cfrac{\sqrt{-555.55\dfrac{\alpha_f d^2}{\overset{\bullet}{m}}C_3}\left[C_4 + e^{x\sqrt{-0.0072\frac{\overset{\bullet}{m}}{\alpha_f d^2}C_3}}\right]}{\left[e^{x\sqrt{-0.0072\frac{\overset{\bullet}{m}}{\alpha_f d^2}C_3}} - C_4\right]} \qquad (11)$$

By employing the boundary condition:

$$T_g(1) - T_g(0) = \Delta T_g \qquad (12a)$$

and the relative Dirichlet condition:

$$T_g(0) = \text{constant} \qquad (12b)$$

the above-mentioned arbitrary constants are identified by solving numerically the following 2×2 non-linear algebraic system:

$$\Delta T_g C_4^2 - \left[\Delta T_g\left(1 + e^{\sqrt{-0.0072\frac{\overset{\bullet}{m}}{\alpha_f d^2}C_3}}\right)\right.$$

$$- 2\sqrt{-555.55\frac{\alpha_f d^2}{\overset{\bullet}{m}}C_3}\left(e^{\sqrt{-0.0072\frac{\overset{\bullet}{m}}{\alpha_f d^2}C_3}} - 1\right)C_4\right] \qquad (13a)$$

$$+ \Delta T_g e^{\sqrt{-0.0072\frac{\overset{\bullet}{m}}{\alpha_f d^2}C_3}} = 0$$

$$C_3 = -\frac{\left[T_g(0)\right]^2 \overset{\bullet}{m} (1-C_4)^2}{555.55\,\alpha_f\,d^2(1+C_4)^2} \qquad (13b)$$

By employing the specific flux of the heat absorbed by the cathode gas, the temperature of the cell, $T_c(x)$, is related to the gas temperature as follows:

$$j(U_t + jr - E) = k(x)(T_c(x) - T_g(x)) \qquad (14)$$

and, therefore, the local thermal conductivity, $k(x)$, can be calculated. The overall heat transfer coefficient k_0 can be estimated as:

$$k_o = \int_0^1 k(x)\,dx = j\int_0^1 \frac{U_t + jr - E}{T_c(x) - T_g(x)}dx \qquad (15)$$

Finally, the mass flow rate can be calculated by using the macroscopic balance for the heat absorbed by gas:

$$\overset{\bullet}{m} = \frac{Q}{c_p\,\Delta T_g} = \frac{\pi dj(U_t + jr - E)}{c_p\,\Delta T_g} \qquad (16)$$

where Q is the overall heat absorbed and c_p is the heat capacity of the atmospheric air.

3. Results and discussion

As it is senseless for a general theoretical study to fix values for thermal diffusivities of the solid electrolyte, the ratio of α_s/α_f is used instead of α_s and it is adjusted to 10000 because it is known that gases are bad heat conductors. Thermal diffusivities of atmospheric air at various temperatures are taken from the appropriate published tables [11].

Figure 1 shows the distribution of the gas temperature along the stack's length for two typical values of temperature difference in the gas phase ($\Delta T_g = 50$ K and $\Delta T_g = 100$ K). The specific effective resistance is 1 Ω and the current density j has been fixed to 100 mA/cm^2. In general, the gas temperature presents its lower values in the inlet area while it increases as gas adsorbs heat along the cell. Obviously, the higher the curves of the gas temperature presented for higher ΔT_g values. Finally, it is worth noticing that the second derivative of $T_g(x)$ is of three to four orders of magnitude lower than the first one in all cases. Additionally, $\dfrac{d^2 T_g(x)}{dx^2}$ is multiplied by the small number α_f and the final product becomes even smaller. This
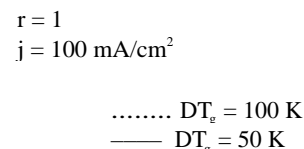
r = 1
j = 100 mA/cm^2

.... DT$_g$ = 100 K
—— DT$_g$ = 50 K

Fig. 1. Profiles of the gas temperature for various ΔT_g.

r = 1 Ω
j = 100 mA/cm^2

.... j = 100 mA/cm^2
———— j = 300 mA/cm^2

r = 1 Ω
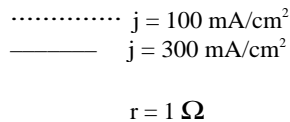
Fig. 2. Dependence of the cell temperature on the current density.

Fig. 3. Spatial distribution of the thermal conductivity.

underlines the negligible character of conduction in the gas phase because of the high Peclet flow of the atmospheric air in the cathode area.

The profile of the cell temperature is presented in Fig. 2 for conditions quite analogous to those of Fig. 1 (j = 100 mA/cm^2 and 300 mA/cm^2, r = 1 Ω). As the gas absorbs the heat produced by the cell, the inlet cathode gas is cooler than the cell and it remains cooler while moving along the cell. Furthermore, it should be stressed that, as $T_c(0) = T_c(1)$, the spatial variation of $T_c(x)$ is always less than 0.5% for any current and resistance (it corresponds to an absolute value of 5 K with respect to the 1000 K of operational temperature), which means that it could be accurately considered as constant, as it has been previously reported in the related bibliography [12]. More precisely, there are some combinations of resistances and currents where the cell temperature presents a really constant profile with maximum absolute divergence from the base line less than 0.0025%. Finally, it should be noted that $T_c(x)$ is independent on ΔT_g as it can obviously derived from eq. (6), (8a) and (8b). It should be also stressed out that in general $T_g(x) \neq T_c(x)$, which can be interpreted as a discontinuity on the gas-solid interface of the gas temperature. In fact, it is a reckless and false conclusion as the interface clearly disappeared by the averaging technique presented above.

Figure 3 presents the spatial profile of the thermal conductivity, k(x), defined by eq. (14) as a function of the dimensionless distance for typical values of specific effective resistance and current density (r = 1 Ω and j = 100 mA/cm^2). In agreement to previously presented results [8],

the thermal conductivity is almost constant for a wide domain along the cell except the inlet and outlet areas, where the end-effects force the convection thermal film to attain its bulk values.

4. Conclusions

The present study presents analytically obtained profiles for both gas and cell temperatures, corresponding to forced convection heat transfer conditions within a hydrogen-fed solid oxide fuel cell. It is found that the temperature profile in the gas phase is monotonically increasing from the inlet to the outlet region while both effective resistance and current density have limited influence on temperature. The cell temperature is found to be practically constant along cell's length, while the overall heat transfer coefficient is favored by the increment of the cell operational temperature in a linear manner.

5. References

[1]   A.V. Virkar, K. Fung, S.C. Singhal, in: DOE Report, DOE/FETC/C-98/7303, 1997.

[2]   N.Q. Minh, T. Takahashi, Science and technology of ceramic fuel cells, Elsevier Science B. V., The Netherlands, 1995.

[3]   M. Bernier, J. Ferguson, R. Herbin, in: Proceedings of the 3rd European Solid Oxide Fuel Cell Forum, Nantes, France, 1998.

[4]   J.O. Bockris, Energy options, Australia & New Zealand Book Co., Sydney, 1980.

[5]   N. Djilali, D. Lu, Int. J. Therm. Sci. **41**, 29 (2002).

[6]   J. Yuan, M. Rokni, B. Sunden, Int. J. Heat Mass Transf. **44**, 4047 (2001).

[7]   S. Dutta, J.H. Morehause, J.A. Khan, Int. J. Hydrogen En. **22**, 883 (1997).

[8]   J. Koh, H. Seo, Y. Yoo, H. Lim, Chem. Eng. J. **3913**, 1 (2001).

[9]   P. Tsiakaras, A.K. Demin, J. Power Sources **102**, 210 (2001).

[10] M. Tenenbaum, H. Pollard, Ordinary Differential Equations, 4th Edition, Dover, New York, 1985.

[11] R.H. Perry, D.W. Green, Chemical Engineers Handbook, 7th Edition, McGraw-Hill, New York, 1997.

[12] C.L. Haynes, W.J. Wepfer, Int. J. Hydrogen En. **26**, 369 (2001).