Model of Adsorption–Reaction–Desorption in a Swarm of Spheroidal Particles

F. A. Coutelieris

National Center for Scientific Research "Demokritos," 15310 Aghia Paraskevi Attikis, Greece

V. N. Burganos and A. C. Payatakes

Institute of Chemical Engineering and High Temperature Chemical Processes, Foundation for Research and Technology, Hellas, and Department of Chemical Engineering, University of Patras, GR 26500, Patras, Greece

The mass transport problem from a Newtonian fluid to a swarm of prolate or oblate spheroidal adsorbing particles under creeping flow conditions is considered here. The spheroidal-in-cell model is used for the analytical description of the flow field within the swarm. A realistic adsorption–reaction–desorption mechanism is used to describe the adsorption of mass on the particle surface, instead of the assumption of instantaneous adsorption that has been adopted previously. The convective diffusion equation accompanied by the appropriate boundary conditions is solved analytically for the case of high Peclet numbers and numerically for the low ones. In both cases, analytical expressions for the overall Sherwood number, the adsorption rate, and the mass transport coefficient were obtained. It was found that the adsorption rate is higher for oblate shapes and for diffusional, instead of convectional, environments. Finally, the assumption of instantaneous adsorption leads to values for the overall Sherwood number and the adsorption efficiency that are 15–50% lower and 10–35% higher, respectively, than those obtained by using the more realistic adsorption–reaction–desorption model. © 2004 American Institute of Chemical Engineers *AIChE J,* 50: 779–785, 2004
Keywords: adsorption, mass transport, spheroid, diffusion, swarm

Introduction

Convective mass transport in swarms of particles is a commonly encountered process in a wide range of industrial and scientific applications such as fluidized beds, separation processes, catalytic and noncatalytic fluid–solid reactions, filters, and so forth. Modeling of fluid flow and of physicochemical processes that take place within particles swarms seems to attract significant scientific interest.

Most of these models concerning creeping or laminar flow within swarms of particles are based on the assumption of spherical or/and cylindrical particles to achieve analytical solutions (Happel, 1958; Kuwabara, 1959; Neale and Nader,

Correspondence concerning this article should be addressed to F. A. Coutelieris at frank@ipta.demokritos.gr.

1974; Tien, 1989). These models are based on the representation of the overall solid mass of the swarm by a spherical or cylindrical solid body, which is embedded in a spherical or cylindrical liquid envelope, respectively. The ratio of the solid volume to the volume of the liquid envelope represents the solid volume fraction of the porous medium. The main advantage of these models is that an analytical expression for the stream function can be obtained demanding significantly smaller effort than that needed for numerical investigations. Such models apply to homogeneous swarms. In practice, a tendency for channeling and/or "bubbling" through the swarm disrupts complete homogeneity. In such cases, one can address homogeneous (or nearly so) subregions of the swarm separately.

In many applications, the particles or grains have shapes that are closer to spheroidal than to spherical (Mou and Howe, 1997). The fluid flow problem within swarm of spheroidal

particles has already been examined either numerically (Ammar and Hsieh, 1991; Epstein and Masliyah, 1972) or analytically (Dassios et al., 1994, 1995), producing accurate results (Burganos et al., 1995). These models are useful in the study of convective mass transport processes in swarms of particles. The basic approach is that of Levich (1962), which applies to mass transport from a fluid stream to a single sphere that adsorbs mass instantaneously. This approximation, which is based on the assumption of a very thin diffusion layer, is valid only for highly convectional processes (Pe $\gg$ 1), and produces analytical expressions for the concentration profile, the overall Sherwood number, and the mass transport coefficient. Later investigations extend Levich's approach by considering mass transport, dominated by convection, through swarms of spherical particles (Pfeffer, 1964; Pfeffer and Happel, 1964; Tardos et al., 1976). Analytical expressions for the overall Sherwood number as a function of the porosity of the swarm were obtained by using both the Kuwabara (1959) and Happel (1958) formalisms for the sphere-in-cell model. Recently, another extension of these models was presented by Coutelieris et al. (1993), in which an analytical solution for the mass transport problem within a swarm of spheroidal particles was obtained for the case where convection dominates diffusion.

Some studies that consider diffusion-dominated [Pe $\leq O(1)$] processes in swarms have also been presented. In this case, it is not possible to obtain an analytical solution even for spherical particles and, thus, numerical treatment is necessary (Masliyah and Epstein, 1972; Prieve and Ruckenstein, 1974; Spielman and Friendlander, 1974). In all these models, the boundary conditions that are imposed in accordance with the Levich approximation have been appropriately modified so as to adequately describe *slow* convective mass transport in sphere-in-cell models. Further modifications have been developed (Coutelieris et al., 1995; Song and Elimelech, 1992) to ensure the continuity of both the concentration and the mass flux on the boundaries.

In all the aforementioned models the particles were assumed to adsorb mass instantaneously (Coutelieris et al., 1993, 1995; Levich, 1962; Pfeffer and Happel, 1964; Song and Elimelech, 1992). It is obvious that the assumption of instantaneous adsorption pertains to a very limited range of applications, given that it is a rare physicochemical phenomenon. A more realistic approach that is adopted here can be based on an adsorption–heterogeneous reaction–desorption mechanism, which describes the adsorption of the diluted mass upon the solid surface with high accuracy (Atkins and DePaula, 2001; Peters et al., 1985; Suzuki and Smith, 1972; Weber and Chakravati, 1974). More precisely, it can be supposed that the solute is first adsorbed on the solid surface, where a heterogeneous reaction takes place and its products, which are supposed to be inactive and of very low concentration, are desorbed into the fluid. The adsorption is assumed to take place on vacant sites that are normally distributed over the solid surface, whereas the overall rate of the process is determined from basic thermodynamics (Smith, 1981).

Theory

The creeping flow of a Newtonian fluid within a porous medium consisting of spheroidal grains can be described by the spheroid-in-cell model (Dassios et al., 1994), which is analo-
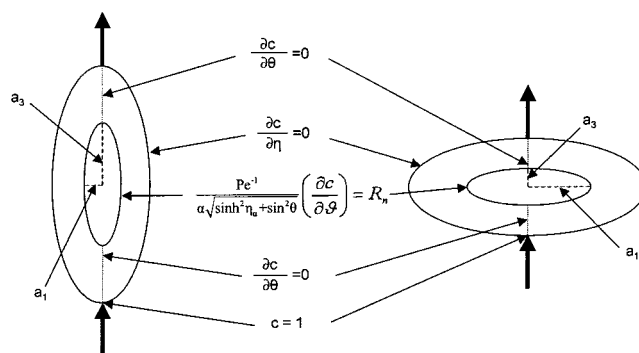

Figure 1. Formulation of the mass transport problem for prolate and oblate shapes.

gous to the previously proposed sphere-in-cell Kuwabara model (Kuwabara, 1959). According to the Dassios et al. (1994) model, the swarm is represented by a solid kernel that is enveloped by a liquid layer, the thickness of which is adjusted so that the porosity of the model is equal to that of the swarm. The flow field is given analytically in terms of an infinite-series expansion (Dassios et al., 1994), the leading term of which has been shown to be a satisfactory approximation (Burganos et al., 1995). Using the leading term, the velocity components for the prolate spheroid-in-cell having long semi-axis a_3 and semifocal distance $\alpha = \sqrt{a_3^2 - 1}$, are given in the prolate coordinates system (η, θ) by

$$u_\eta = \frac{-1}{\alpha^2 \sqrt{\sinh^2\eta + \sin^2\theta} \, \sinh\eta \sin\theta} \frac{\partial \Psi}{\partial \theta} \tag{1a}$$

$$u_\theta = \frac{1}{\alpha^2 \sqrt{\sinh^2\eta + \sin^2\theta} \, \sinh\eta \sin\theta} \frac{\partial \Psi}{\partial \eta} \tag{1b}$$

and the stream function is given by

$$\Psi(\eta, \theta) = \frac{\alpha}{D} \left\{ \Lambda_2 G_2(\cosh\eta) + \Lambda_3 \left[\frac{5G_4(\cosh\eta_\beta)}{G_1(\cosh\eta_\beta)} G_1(\cosh\eta) \right. \right.$$
$$\left. \left. + G_4(\cosh\eta) \right] + \Lambda_4 H_2(\cosh\eta) \right\} G_2(\cos\theta) \tag{2}$$

where D, Λ_2, Λ_3, and Λ_4 are η- and θ-dependent coefficients defined by Dassios et al. (1994) and $G_n(x)$ and $H_n(x)$ are the Gegenbauer polynomials of the first and second kind, respectively, of degree $-(1/2)$ and of order n.

By assuming constant diffusivity, the governing equation for the steady state mass transport of the solute A, under creeping flow conditions (see Figure 1), can be written in dimensionless form as

$$\mathbf{u}\nabla c_A = \text{Pe}^{-1}\nabla^2 c_A \tag{3}$$

where $\mathbf{u}$ is the velocity vector and Pe is the Peclet number. Equation 3 can be expressed in prolate spheroidal coordinates as (Coutelieris et al., 1995)

$$u_\eta \frac{\partial c_A}{\partial \eta} + u_\theta \frac{\partial c_A}{\partial \theta} = \frac{\text{Pe}^{-1}}{\alpha \sqrt{\sinh^2\eta + \sin^2\theta}}$$

$$\times \left(\frac{\partial^2 c_A}{\partial \eta^2} + \coth \eta \frac{\partial c_A}{\partial \eta} + \frac{\partial^2 c_A}{\partial \theta^2} + \cot \theta \frac{\partial c_A}{\partial \theta} \right) \quad (4)$$

This equation is to be integrated with the following boundary conditions (see also Figure 1)

$$c_A(\eta = \eta_\beta,\ \theta = \pi) = 1 \tag{5a}$$

$$\left. \frac{\partial c_A}{\partial \eta} \right]_{\eta = \eta_\beta} = 0 \quad 0 \le \theta < \pi \tag{5b}$$

$$\left. \frac{\partial c_A}{\partial \theta} \right]_{\theta = \pi} = 0 \quad \eta_\alpha \le \eta \le \eta_\beta \tag{5c}$$

$$\left. \frac{\partial c_A}{\partial \theta} \right]_{\theta = 0} = 0 \quad \eta_\alpha \le \eta \le \eta_\beta \tag{5d}$$

and

$$\frac{\text{Pe}^{-1}}{\alpha \sqrt{\sinh^2\eta_\alpha + \sin^2\theta}} \left[\frac{\partial c_A}{\partial \eta} \right]_{\eta = \eta_\alpha} = R_n \quad 0 < \theta < \pi \tag{5e}$$

The boundary condition described by Eqs. 5a and 5b was proposed by Coutelieris et al. (1995) to ensure the continuity of the concentration on the outer boundary of the cell for any Peclet number. Equations 5c and 5d express the axial symmetry that has been assumed for the system.

The last boundary condition describes a typical adsorption/reaction/desorption mechanism for the component A on the solid surface (Atkins and DePaula, 2001; Smith, 1981). R_n in Eq. 5e denotes the overall adsorption rate, given by

$$R_n = k_s c_{AS}^n \tag{6}$$

where k_s is the rate constant of the heterogeneous reaction on the surface. The concentration of component A on the solid surface, c_{AS}, is calculated by solving the nonlinear equation

$$k_s c_{AS}^n + [k_A^d + k_A^a c_A(\eta_\alpha, \theta) N] c_{AS} - k_A^a c_A(\eta_\alpha, \theta) \xi_m = 0 \tag{7}$$

The above equation correlates the hard-to-measure surface concentration of A, c_{AS}, with its concentration in the bulk phase very close to the solid surface, $c_A(\eta_\alpha, \theta)$ by considering the balance for the active sites on the adsorbing surface. The terms k_A^a and k_A^d denote the adsorption and desorption rate constants of component A, respectively; ξ_m is the concentration of the vacant sites of the solid surface; and N is Avogadro's number; whereas n denotes the order of the heterogeneous reaction. In general, only the cases of $n = 0$, $n = 1$, and $n = 2$ are of practical interest, whereas zeroth order reactions have few applications (Atkins and DePaula, 2001). Therefore, the present investigation deals only with first- and second-order heterogeneous reactions. Finally, it should be noted that, for

$n = 2$, the boundary condition 5e introduces nonlinearity in the system and thus makes it necessary to use an iterative technique to obtain the solution.

The case of high Peclet numbers

For the case of high Peclet numbers (Pe $\gg$ 1), the concentration boundary layer is very thin compared to the local radius of curvature of the particle, and, thus, the curvature term $\coth \partial c_A/\partial \theta$ can be neglected along with the tangential diffusion terms $\partial^2 c_A/\partial \theta^2$ and $\cot \partial c_A/\partial \theta$, which have been shown to be negligible for high Peclet values (Coutelieris et al., 1995). With these simplifications, Eq. 3 becomes parabolic on θ and it can be solved analytically in a manner quite similar to that of Coutelieris et al. (1993), providing the following concentration profile in the fluid phase

$$c_A(z) = c_2 \int_0^z e^{-(4/9)t^3} dt + c_3 \tag{8}$$

where

$$z = \sqrt[3]{\frac{\alpha E \text{Pe}}{4D(\sinh^2\eta_\alpha - 1)}}\ \eta f(\theta) \tag{9}$$

The integration constants c_2 and c_3 are to be calculated by solving the nonlinear system

$$\sqrt[3]{\frac{E}{4\alpha^5 \text{Pe}^2}}\ \frac{f(\theta)}{\sqrt{\sinh^2\eta_\alpha + \sin^2\theta}}\ c_2 - R_n(c_3) = 0 \tag{10a}$$

$$1.17 c_2 + c_3 = 1 \tag{10b}$$

where E and $f(\theta)$ are terms defined in Coutelieris et al. (1993) as follows:

$$E = \frac{\Lambda_2}{2} + \frac{15\Lambda_3}{4}\cosh^2(\eta_\alpha) - \frac{3\Lambda_3}{4}$$

$$+ \frac{\Lambda_4}{4}\ln\frac{\cosh(\eta_\alpha) + 1}{\cosh(\eta_\alpha) - 1} - \frac{\Lambda_4}{2}\frac{\cosh(\eta_\alpha)}{\cosh^2(\eta_\alpha) - 1} \tag{10c}$$

$$f(\theta) = \frac{\sqrt[3]{2}\ \sin\theta}{\sqrt[3]{\sin\theta\cos\theta - \theta}} \tag{10d}$$

It should be noted that coefficient c_3 represents the concentration of A in the fluid very close to the solid surface according to the relation

$$\lim_{\eta \to \eta_\alpha} c_A = \lim_{z \to 0} c_A = c_3 \tag{11}$$

Some mathematical manipulations lead to the following expressions for the overall Sherwood number Sh_o

$$\text{Sh}_o = \frac{3.383}{2\pi\left(1 + \dfrac{a_3}{\alpha}\sin^{-1}\dfrac{\alpha}{a_3}\right)}\sqrt[3]{E\text{Pe}}\int_0^{2\pi}f(\theta)d\theta \qquad (12)$$

whereas the overall adsorption efficiency λ_o, defined as the ratio of the rate of the adsorption over the rate of the upstream flux, is given by

$$\lambda_o = \frac{1.26}{\alpha^2\sinh^2\eta_\beta}\sqrt[3]{\frac{E}{\text{Pe}}}\int_0^{2\pi}c_2 f(\theta)d\theta \qquad (13)$$

Note that the overall adsorption efficiency depends on the adsorption–reaction–desorption phenomena that take place on the solid surface, whereas the overall Sherwood number is independent of them because of the high concentration gradients caused by convection.

The corresponding expressions for the case of oblate spheroids-in-cell are similar to those of the prolate case, presenting differences only in the following equations

$$\sqrt[3]{\frac{E}{4\alpha^5\text{Pe}^2}}\frac{f(\theta)}{\sqrt{\cosh^2\eta_\alpha - \sin^2\theta}}c_2 - R_n(c_3) = 0 \qquad (10a')$$

$$\text{Sh}_o = \frac{3.383}{2\pi\left(1 + \dfrac{a_3^2}{2\alpha}\ln\dfrac{1+\alpha}{1-\alpha}\right)}\sqrt[3]{E\text{Pe}}\int_0^{2\pi}f(\theta)d\theta \qquad (12')$$

$$\lambda_o = \frac{1.26}{\alpha^2\cosh^2\eta_\beta}\sqrt[3]{\frac{E}{\text{Pe}}}\int_0^{2\pi}c_2 f(\theta)d\theta \qquad (13')$$

The case of moderate and low Peclet numbers

In this case, Eq. 4 should be solved without any simplification and thus a numerical approach is indispensable for the determination of the concentration c_A. A finite-differences discretization scheme was used for the integration of Eq. 4 along with Eqs. 5a–5e. The grid spacing is chosen to be nonuniform because it has been proved that appropriately nonuniform discretization performs better than the equal-spaced one (Sun and Levan, 1995). The overall Sherwood number Sh_o for the case of prolate spheroids is given by

$$\text{Sh}_o = \frac{-\alpha\sinh\eta_\alpha}{1 + \dfrac{a_3}{\alpha}\sin^{-1}\dfrac{\alpha}{a_3}}\int_0^{\pi}\frac{\sin\partial\left(\dfrac{\partial c_A}{\partial\theta}\right)_{\eta=\eta_\alpha}}{c_A(\eta_\beta,\theta) - c_A(\eta_\alpha,\theta)}d\theta \qquad (14)$$

and for the case of oblate spheroids by

$$\text{Sh}_o = \frac{-\alpha\cosh\eta_\alpha}{1 + \dfrac{a_3^2}{2\alpha}\ln\dfrac{1+\alpha}{1-\alpha}}\int_\pi^0\frac{\sin\theta\left(\dfrac{\partial c_A}{\partial\eta}\right)_{\eta=\eta_\alpha}}{c_A(\eta_\beta,\theta) - c_A(\eta_\alpha,\theta)}d\theta \qquad (14')$$

Consequently, the overall adsorption efficiency λ_o is given by

$$\lambda_o = \frac{-2\alpha\sinh\eta_\alpha}{\text{Pe}\alpha^2\sinh^2\eta_\beta}\int_0^{\pi}\sin\theta\left(\frac{\partial c_A}{\partial\eta}\right)_{\eta=\eta_\alpha}d\theta \qquad (15)$$

and

$$\lambda_o = \frac{2\alpha\sinh\eta_\alpha}{\text{Pe}\alpha^2\cosh^2\eta_\beta}\int_0^{\pi}\sin\theta\left(\frac{\partial c_A}{\partial\eta}\right)_{\eta=\eta_\alpha}d\theta \qquad (15')$$

for the cases of prolate and oblate spheroids, respectively.

Results and Discussion

Figure 2 shows the concentration profiles for component A at different angular positions for the cases of prolate (a) and oblate (b) spheroids-in-cell, in the case of the first-order heterogeneous reaction ($n = 1$). In the case of high Peclet numbers (say, Pe = 1000), the concentration presents higher slopes for the case of prolate spheroids than that for oblate ones, given that it has also been observed previously for the case of instantaneous adsorption (Coutelieris et al., 1993). In general, the concentration approaches its bulk value at small distances from the solid surface. The dashed lines in Figure 2 denote the concentration profiles for a low Pe value (Pe = 15), selected so as to ensure that the condition $\partial c_A/\partial\eta = 0$ at $\eta = \eta_\beta$ and $\theta = 0$ holds (Coutelieris et al., 1995). A sharper decrease of the concentration gradients towards the solid surface is observed, compared with that for the case of high Peclet numbers, because diffusion dominates convection as Pe becomes small. An interesting weakness of the Levich approach arises near $\eta = \eta_\beta$ within the tail region ($\theta \to \pi$), where the Levich model gives very small concentration values, whereas these values in reality might be comparable to the bulk value. Levich's approach fails to predict acceptable concentration values in this area because the fundamental assumption of a very thin boundary concentration layer is not valid in this region even for high Pe values. Finally, it should be noted that the concentration on the surface, $c_A(\eta_\alpha, \theta)$, presents maximum at the cell-frontal point ($\eta = \eta_\beta$, $\theta = 0$) and a monotonic decrease is observed as θ tends to π for the case of prolate spheroids and high Pe values. The maximum concentration value appears near the equator for the case of oblate spheroids, where the thickness of the diffusion film becomes minimum (Coutelieris et al., 1995). This behavior is not observed for small Pe values because in such cases surface concentration values are much larger than those of the large Pe case, and so the accessibility of adsorbent to the solid surface is sufficiently large to produce nearly uniform concentration profiles for the adsorbed mass.

The effect of the spheroidal shape on the overall Sherwood number is presented in Figure 3 for the case of high (a) and low (b) Peclet values. A monotonic decrease of Sh_o, with increasing aspect ratio a_3, is observed for all cases because prolate spheroids present higher diffusional resistance than that of oblate ones, as reported by Coutelieris et al. (1993). Oblate spheroids present larger impact surface than that of prolate ones, and thus their capacity for adsorption is higher. This advantage of oblate shapes becomes very
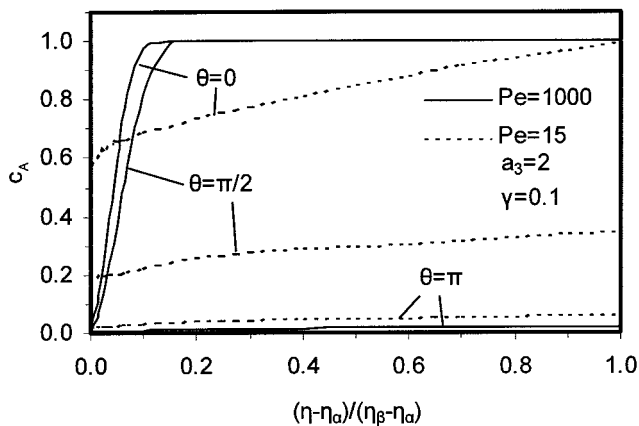
(a)
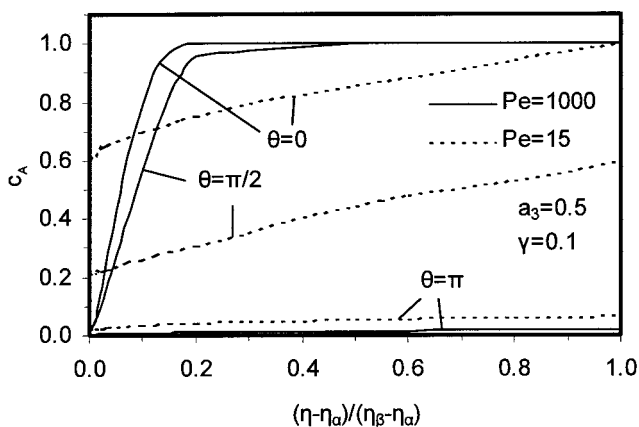
(b)

Figure 2. Concentration profiles for prolate (a) and oblate (b) spheroidal shapes for a high and a low Peclet number at three different angular positions.

weak in the case of small Pe values because almost all the parts of the adsorbing solid surface become active as diffusion becomes dominant. The overall Sherwood number depends appreciably on the order of the heterogeneous reaction in the case of small Peclet values, unlike the case of large ones, because the concentration gradients become less significant. In this case, a decrease of Sh_o is observed when the reaction is of second-order (rather than of first-order) because a higher reaction order causes lower concentration gradients and larger amounts of adsorbed mass. The overall Sherwood number can be 15–50% higher, depending on Pe, the shape, the porosity, and the order of the reaction, compared to the values obtained assuming instantaneous adsorption for either high or low Peclet values (Coutelieris et al., 1993, 1995). This occurs because the concentration on the solid surface attains nonzero values and, thus, the difference $c_A(\eta_\beta, \theta) - c_A(\eta_\alpha, \theta)$ becomes almost nil for some θ-values. The decrease of this driving force is significantly larger than the decrease of the concentration gradients, which is ob-

served when a realistic adsorption process is assumed instead an instantaneous one.

The dependency of the overall adsorption efficiency on the Peclet number for both prolate (a) and oblate (b) shapes is given in Figure 4a and 4b, where the heterogeneous reaction is assumed to be either first-order ($n = 1$) or second-order ($n = 2$). A significant decrease of the adsorption efficiency occurs as Pe increases. The values for the overall adsorption efficiency obtained using the instantaneous adsorption model are 10–35% higher than those obtained here because of the higher concentration gradients that instantaneous adsorption generates.

Figure 5 shows the overall mass transport coefficient k_o as a function of the aspect ratio a_3, for constant volume (a) and for constant surface (b) of the solid spheroid for a large (Pe = 1000) and a medium-to-small (Pe = 15) Peclet value by assuming first-order reaction. The solid volume fraction of the cell was kept at a fixed value ($\gamma = 0.1$). The overall mass transport coefficient is chosen to express the rate of mass transfer because Sh_o depends on the short semiaxis (characteristic length) of the spheroid, which cannot be kept constant for both the iso-surface and the iso-volume case. Given a high Pe value, the consumption of the component A on the solid surface is significantly larger for swarms that consist of oblate spheroids than those consisting of spheres or prolate spheroids.
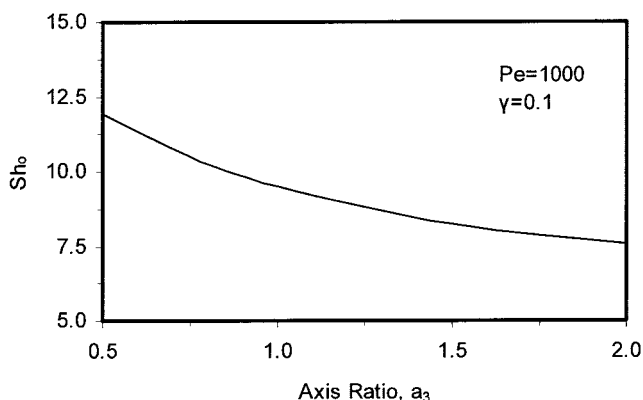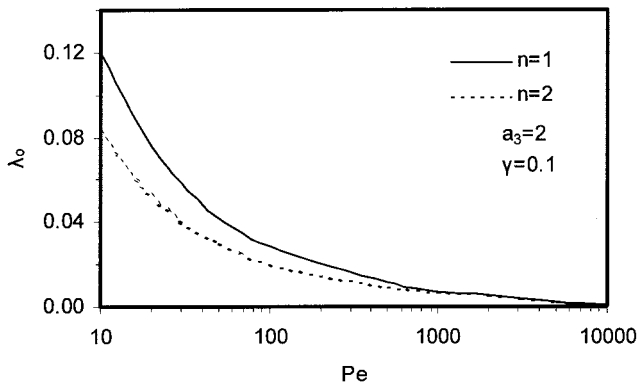

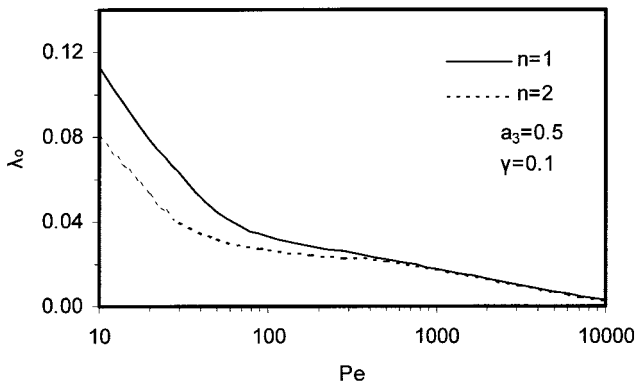**(a)**
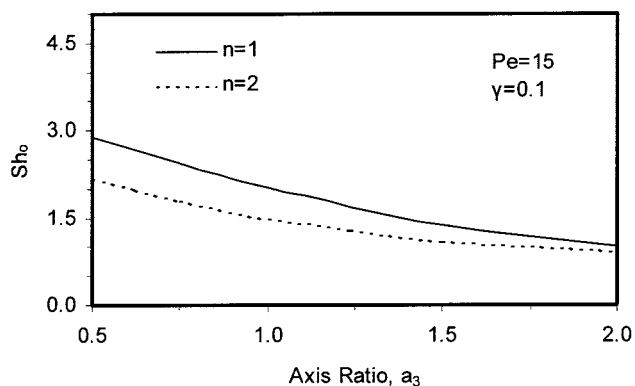
(b)

Figure 3. Dependency of the overall Sherwood number on the axis ratio of the spheroid for a high (a) and a low (b) Peclet number.

(a)

(b)

Figure 4. Influence of the Peclet number on the adsorption efficiency for prolate (a) and oblate (b) spheroids-in-cell.

In the constant volume case, in particular, the mass transfer rate is almost doubled by changing the aspect ratio from 2 to 0.5, whereas its effect on k_o is less drastic but still significant when the surface area of the solid spheroid is kept constant. Smaller Pe values give significantly smaller mass transfer coefficients, which have a smoother behavior as the aspect ratio varies, compared with those of large Pe values.

Conclusions

The problem of mass transfer from a moving Newtonian fluid to a swarm of prolate and/or oblate stationary spheroidal adsorbing particles under creeping flow conditions is solved using a spheroidal-in-cell model. The flow field through the swarm was obtained by using the spheroid-in-cell model proposed by Dassios et al. (1994). The leading term of the series expansion of the stream function is used to approximate the velocity components. An adsorption–reaction–desorption scheme is used to provide the boundary conditions on the surface of the spheroid. The convective diffusion equation is solved analytically for the case of large Peclet numbers, and the overall Sherwood number, the adsorption rate, and the mass transport coefficient were obtained in analytical form. For the case of small Pe values a

finite-difference scheme with nonuniform grid is used to solve the problem numerically. It was found that the mass transfer rate is higher for oblate spheroids-in-cell than it is for spheres-in-cell and prolate spheroids-in-cell, assuming constant volume and surface area especially for large Peclet values. Consequently, the oblate shape offers a significant advantage over that of spheres and prolate spheroids, especially for large Pe values, in which case convection is dominant. This advantage weakens considerably for small Pe values when diffusion becomes dominant. Taking into account the finite rates of adsorption, reaction, and desorption gives an overall Sherwood number 15–50% smaller than that obtained by assuming instantaneous adsorption and reaction.
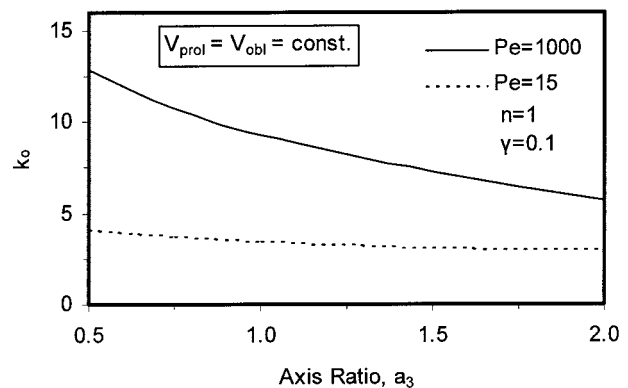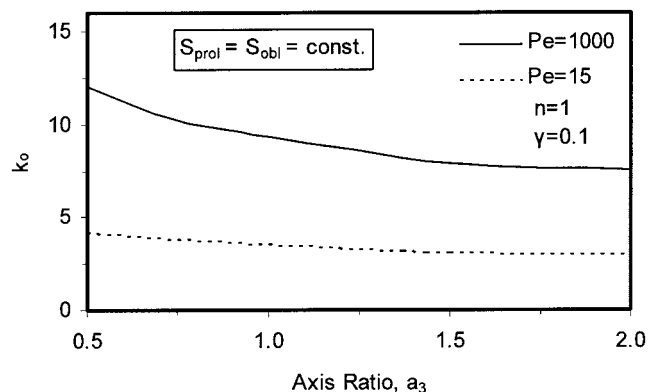
Notation
Latin letters

a_1, a_3 = semiaxes of the solid spheroid

(a)

(b)

Figure 5. Dependency of the mass transport coefficient on the axis ratio for spheroids-in-cell of the same volume of the solid phase (a) and of the same solid surface area (b) and the same porosity.

c_2, c_3 = coefficients from the integration of Eq. 8
c_A = concentration of component A
c_{AS} = concentration of component A on the solid surface
D = determinants defined in Dassios et al. (1994)
E = coefficient defined by Coutelieris et al. (1993)
$f(\theta)$ = function defined by Coutelieris et al. (1993)
k_s = rate constant of the heterogeneous reaction
k_A^a, k_A^d = adsorption and desorption rate constants
N = Avogadro's number
n = order of the heterogeneous reaction
Pe = Peclet number ($= U_\infty a_1/D_{AB}$), where U_∞ is the approach velocity and D_{AB} is the diffusion coefficient
R_n = overall adsorption rate
Sh_o = overall Sherwood number ($= k_o a_1/D_{AB}$), where k_o is the overall mass transport coefficient and D_{AB} is the diffusion coefficient
$\mathbf{u}$ = fluid velocity vector
u_η, u_θ = η- and θ-components of the fluid velocity, respectively
z = dependent variable defined by Eq. 9

Greek letters

α = semifocal distance
η = spheroidal coordinate
η_α, η_β = values of η in the inner and the outer spheroidal surface, respectively
θ = spheroidal coordinate
λ_o = overall adsorption efficiency
$\Lambda_2, \Lambda_3, \Lambda_4$ = coefficients defined in Dassios et al. (1994)
ξ_m = site concentration ($=$ vacant sites per unit surface)
Ψ = stream function

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