Defect and Diffusion Forum Vols. 297-301 (2010) pp 1420-1425
Online available since 2010/Apr/13 at www.scientific.net

doi:10.4028/www.scientific.net/DDF.297-301.1420

On the Significance of Flow Regime for the Heat Transfer in Solid Oxide Fuel Cells

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Keywords: Fuel cell, heat transfer, conduction, convection, mathematical modeling.

Abstract. The steady state heat transfer that takes place in a hydrogen-fed tubular Solid Oxide Fuel Cell is considered here. The heat is produced due to the electrochemical reaction of the hydrogen that feds the cell with oxygen anions. An averaging technique is used to formulate a relatively simple one-dimensional heat transfer problem. The conduction-convection equation describing the heat transfer from the electrolyte's surface to the moving gas that surrounds the cell's cathode is solved analytically under the assumption of iso-thermal conditions. Three different cases are considered for the flow of the cathode gas: (a) plug flow, (b) fully developed incompressible laminar flow, and (c) compressible flow. Analytical expressions for the spatial distribution of gas and cell temperature along the cell's length are obtained. For constant mass flow rate, different flow regimes produce almost the similar spatial distributions for the gas temperature and, consequently, the consideration of the flow regime is of low importance in the design of fuel cell stacks.

Introduction

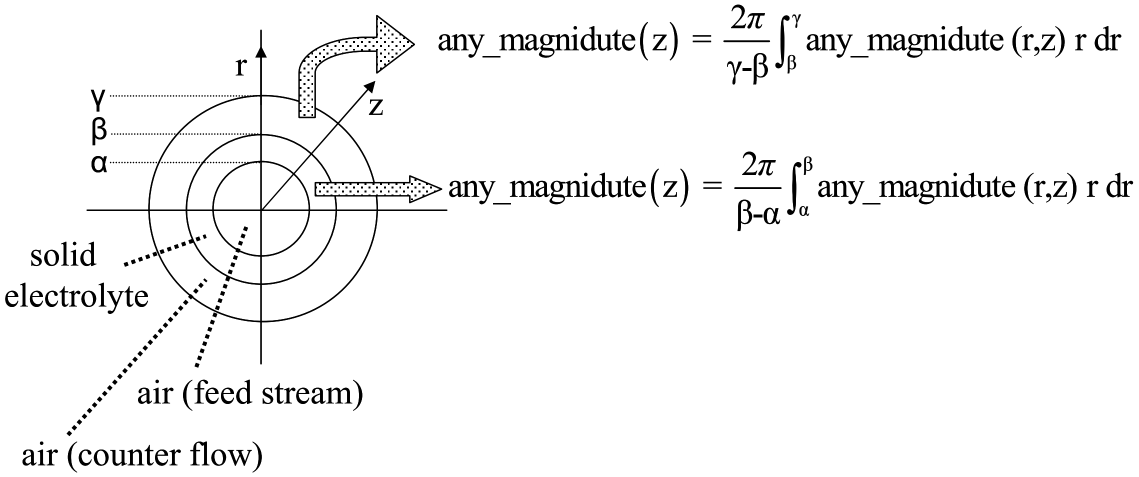
This work can be considered as a direct exploitation of our previous study for the thermal transport phenomena occurred within a solid oxide fuel cell operating at high temperatures [1]. The analytical model presented there, takes into account the complete conduction-convection heat transfer equation along the cell, where it was assumed that the atmospheric air flows in the cathode compartment under fully developed incompressible laminar flow conditions. Here, we extend this limitation by assuming three different flow regimes of the cathode gas: (a) plug flow, (b) fully developed incompressible laminar flow, and (c) compressible flow. By using the averaging technique presented in detail elsewhere [1], the original 2-D heat transfer problem is reduced to a simplified 1-D formulation, which is analytically solved here. Therefore, analytical expressions for the spatial distribution of the cell and gas temperatures could be obtained.

Formulation of the problem

A typical fuel cell consists of an anode and a cathode part between which a catalyst (electrolyte) layer exists. The atmospheric air flows in the cathode gas channel while a hydrogen-rich mixture flows under various conditions at the anode channel. The present model studies a typical cylindrical fuel cell, presented in detail elsewhere [1-3]. By considering a circular ring of the domain of the interest, it is easy to define an averaging procedure in a cross section z, as it is presented in Fig. 1 [1]. In that case, the gas temperature is defined as follows

$$T_g(z) = \frac{1}{\beta-\alpha}\int_{\alpha}^{\beta} T_g(r,z)2\pi r dr \qquad (1)$$

where $T_g(r,z)$ is the original two dimensional gas temperature.



$$\text{any\_magnidute}(z) = \frac{2\pi}{\gamma-\beta}\int_{\beta}^{\gamma} \text{any\_magnidute }(r,z)\; r\, dr$$

$$\text{any\_magnidute}(z) = \frac{2\pi}{\beta-\alpha}\int_{\alpha}^{\beta} \text{any\_magnidute }(r,z)\; r\, dr$$

Figure 1 - The averaging technique

The heat transfer in solid phase

As the conduction is the only available mechanism for the heat transport within solids, the heat transfer in the solid phase of the cell is described by the following differential equation

$$-\alpha_s \frac{d^2T_c(x)}{dx^2} = j\,(U_t + j\, r_{eff} - E(x)\,) \tag{2}$$

which has been analytically solved in [1]. It was found that

$$T_c(x) = a_0x^2 + a_1 \frac{(1-2\eta_f x)}{2\eta_f^2} \ln[\, p_{H_2}(0)(1-\eta_f x)\,] -$$

$$- a_1 \frac{(1-p_{H_2}(0))^2(1-\eta_f x))}{2\,[p_{H_2}(0)]^2\eta_f^2} \ln[1-p_{H_2}(0)(1-\eta_f x)\,] +$$

$$+ \frac{a_1}{2} x^2 \ln[\frac{p_{H_2}(0)(1-\eta_f x)}{1-p_{H_2}(0)(1-\eta_f x)}\;] + C_{s1}x + C_{s2} \tag{3}$$

where the boundary conditions

$T_c(1) = T_c(0)$ (4a)

and

$T_c(0) =$ constant (finite value) (4b)

impose that

$$C_{s1} = \frac{a_1 \ln[\, p_{H_2}(0)]}{2\eta_f^2} - \frac{a_1(1-2\eta_f)}{2\eta_f^2} \ln[\, p_{H_2}(0)(1-\eta_f)\,] - a_0 -$$

$$- \frac{a_1(1-p_{H_2}(0))^2}{2\,[p_{H_2}(0)]^2\eta_f^2} \ln[1-p_{H_2}(0)] + a_1 \frac{(1-p_{H_2}(0))^2(1-\eta_f)}{2\,[p_{H_2}(0)]^2\eta_f^2} \ln[1-p_{H_2}(0)(1-\eta_f)\,]$$

$$- \frac{a_1}{2} \ln[\frac{p_{H_2}(0)(1-\eta_f)}{1-p_{H_2}(0)(1-\eta_f)}\;] \tag{5a}$$

$$C_{s2} = T_c(0) - \frac{a_1 \ln[\, p_{H_2}(0)]}{2\eta_f^2} + \frac{a_1(1-p_{H_2}(0))^2}{2\,[p_{H_2}(0)]^2\eta_f^2} \ln[1-p_{H_2}(0)] \tag{5b}$$

The heat transfer in gas phase

The governing differential equation for the heat transfer in the gas phase can be written as

$$u(x)\frac{dT_g(x)}{dx}=\alpha_f\frac{d^2T_g(x)}{dx^2} \tag{6}$$

where u(x) is the gas velocity. Obviously, the solution of the above equation depends on the velocity profile. Three different cases are considered for the flow of the cathode gas: plug flow, fully developed incompressible laminar flow and compressible flow.

By assuming plug flow regime, i.e. that u(x) = u = const., the differential eq. (6) becomes

$$u\frac{dT_g(x)}{dx}=\alpha_f\frac{d^2T_g(x)}{dx^2} \tag{7}$$

having a general solution of the form

$$T_g(x)=C_{g1}\frac{\alpha_f}{u}e^{\frac{\alpha_f}{u}x}+C_{g2} \tag{8}$$

By employing the boundary conditions

$$T_g(0) - T_g(1) = \Delta T_g \tag{9a}$$

and

$$T_g(0) = \text{constant (finite value)} \tag{9b}$$

the above-mentioned arbitrary constants are identified as

$$C_{g1}=\frac{\alpha_f}{u\left(1-e^{\frac{\alpha_f}{u}}\right)}\Delta T_g \tag{10a}$$

and

$$C_{g2}=T_g(0)-\frac{\Delta T_g}{1-e^{\frac{\alpha_f}{u}}} \tag{10b}$$

The solution for the case of fully developed incompressible laminar flow has already been presented in detail elsewhere [1], thus we make use here of the final form of the solution

$$T_g(x)=\frac{\Delta T_g}{1-e^{\frac{4\dot{m}}{\pi d^2\alpha_f}}}\left(e^{\frac{4\dot{m}}{\pi d^2\alpha_f}x}-1\right)+T_g(0) \tag{11}$$

where $\dot{m}$ is the constant mass flow rate of the gas through a circular surface of diameter d. It is worth noticing that the viscosity of the atmospheric air (gas mixture) should be constant in order to use the Hagen-Poiseuille concept for the description of the velocity, but viscosity is in general an unknown function of the gas temperature. However, it has been reported that the variation of air viscosity is less than 4% per 25 K [4] and, thus, it can be considered as constant for variation of gas temperature up to 100 K. On the other hand, high temperature differences may damage the material of the solid electrolyte and this is another reason for keeping the temperature differences up to 100 K.

To address the case of compressible flow, it is considered that the moving air is a compressible ideal gas, thus by assuming constant mass flow rate of the gas, $\dot{m}=\rho\left(\pi d^2/4\right)u$, through a circular surface of diameter d, the velocity u(x) can be expressed as a function of gas temperature as follows

$$u(x)=\frac{V_A^0\dot{m}}{\pi\frac{d^2}{4}MW_{air}}T_g(x)=0.0036\frac{\dot{m}}{d^2}T_g(x) \tag{12}$$

thus, the differential equation describing the heat transfer in the gas phase becomes:

$$0.0036\frac{\dot{m}}{d^2}T_g(x)\frac{dT_g(x)}{dx}=\alpha_f\frac{d^2T_g(x)}{dx^2} \qquad (13)$$

The solution of the above equation can be obtained by using a downgrading technique for non-linear differential equations [5] and is of the form

$$T_g(x)=\frac{\sqrt{-555.55\frac{\alpha_f d^2}{\dot{m}}C_{g1}}\left[C_{g2}+e^{x\sqrt{-0.0072\frac{\dot{m}}{\alpha_f d^2}C_{g1}}}\right]}{\left[e^{x\sqrt{-0.0072\frac{\dot{m}}{\alpha_f d^2}C_{g1}}}-C_{g2}\right]} \qquad (14)$$

By employing the boundary conditions described previously, the above-mentioned arbitrary constants are identified by solving numerically the following 2x2 non-linear algebraic system

$$\Delta T_g C_{g2}^2 - \left[\Delta T_g\left(1+e^{\sqrt{-0.0072\frac{\dot{m}}{\alpha_f d^2}C_{g1}}}\right)-2\sqrt{-555.55\frac{\alpha_f d^2}{\dot{m}}C_{g1}}\left(e^{\sqrt{-0.0072\frac{\dot{m}}{\alpha_f d^2}C_{g1}}}-1\right)\right]C_{g2} +$$

$$+\Delta T_g e^{\sqrt{-0.0072\frac{\dot{m}}{\alpha_f d^2}C_{g1}}}=0 \qquad (15a)$$

$$C_{g1} = -\frac{\left[T_g(0)\right]^2\ \dot{m}\ (1-C_{g2})^2}{555.55\ \alpha_f\ d^2\ (1+C_{g2})^2} \qquad (15b)$$

Solution technique

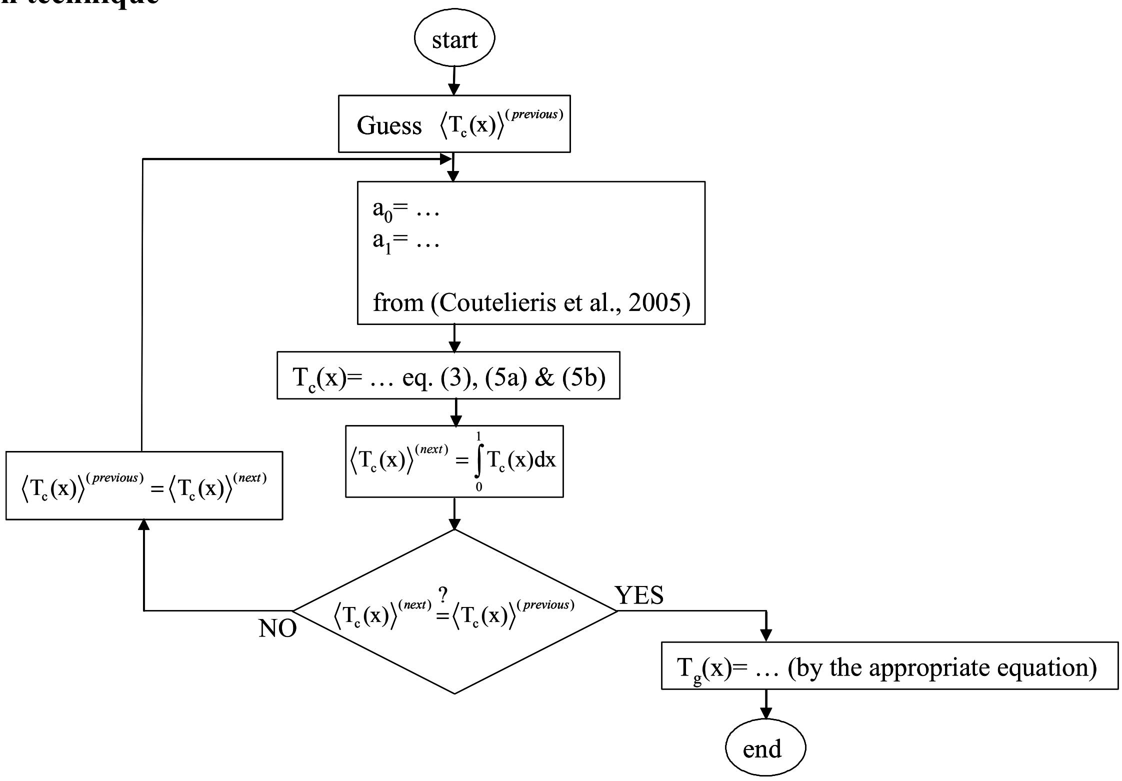


Figure 2 - Iterative scheme for the solution

The above Fig. 2 presents the generalized iterative scheme used for the solution of the problem. In general, the algorithm is as follows

STEP1: Guess an initial value for $\langle T_c(x)\rangle^{(previous)}$

STEP2: Calculate a_0 and a_1 used in eq. (3) [1]

STEP3: Estimate $T_c(x)$ from eq. (3), (5a) & (5b)

STEP4: Find $\langle T_c(x)\rangle^{(next)} = \int_0^1 T_c(x)dx$

STEP5: If $\langle T_c(x)\rangle^{(next)} = \langle T_c(x)\rangle^{(previous)}$ stop the procedure since the solution is $T_c(x)$ estimated at STEP2 ang go to STEP6; otherwise set $\langle T_c(x)\rangle^{(previous)} = \langle T_c(x)\rangle^{(next)}$ and repeat the procedure between STEP2 to STEP5.

STEP6: Assume a specific type for the gas flow and calculate $T_g(x)$ from the appropriate equations

A FORTRAN code has been developed in order to implement the above algorithm. As it is senseless for a general theoretical study to discuss the specific values for the parameters used, typical estimations/measurements are taken from the literature [4].

Results and discussion

The profile of the cell temperature is presented in Fig. 3 for typical values of current density (100mA/cm^2), specific effective resistance (1 Ωcm^2) and temperature difference in the gas phase (ΔT_g = 100 K). As $T_c(0) = T_c(1)$, the spatial variation of $T_c(x)$ is always less than 0.5% for any current and resistance (it corresponds to an absolute value of 5 K with respect to the 1000 K of operational temperature), which means that it could be accurately considered as constant, as it has been previously reported in the related bibliography [6]. More precisely, there are some combinations of resistances and currents where the cell temperature presents a really constant profile with maximum absolute divergence from the base line less than 0.0025%. Finally, it should be noted that $T_c(x)$ is independent on ΔT_g. It should be also stressed out that, in general, $T_g(x) \neq T_c(x)$, which can be interpreted as a discontinuity on the gas-solid interface of the gas temperature. In fact, it is a reckless and false conclusion as the interface clearly losses its physical meaning after applying the averaging procedure presented above.

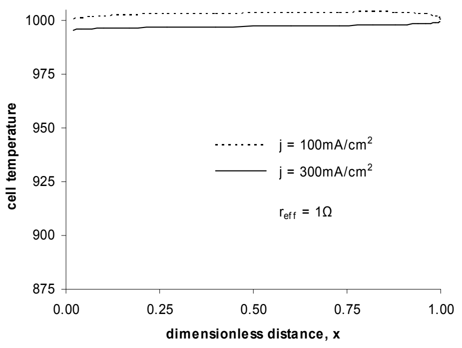


Figure 3 - Spatially distributed profile of the cell temperature

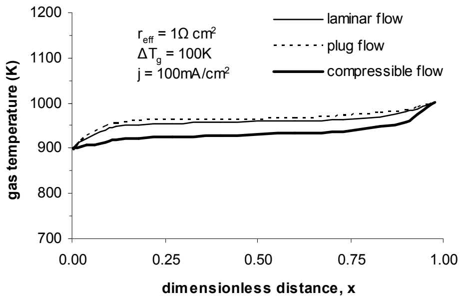


Figure 4 - Spatial profile of the gas temperature for various flow conditions.

The distribution of the gas temperature along the dimensionless length of the cell is presented in Fig. 4 for a typical value of current density (j=100mA/cm^2). Various flow regimes (plug flow, fully developed incompressible laminar flow, compressible flow) have been considered. The specific effective resistance (defined as $r_{eff} = r_{ohm}L + r_{\eta a} + r_{\eta c}$, where r_{ohm} is the ohmic resistance and $r_{\eta a}$ and $r_{\eta c}$ are the polarization anodic and cathodic resistances, respectively) is 1 Ωcm^2 and the overall temperature difference in the gas phase, ΔT_g, has been fixed to 100 K. This value for the temperature difference has been chosen to be low enough to ensure that the viscosity does not change significantly and, therefore, the Hagen-Poiseuille flow regime remains valid. The plug flow regime generates almost the same spatial distribution for the gas temperature as that produced by assuming laminar flow. This is because the second Euclidean norm (magnitude) of the velocity vector was constant in all cases. On the other hand, a small decrement of the gas temperature is observed when compressible flow is assumed because the magnitude of the velocity in that case is more than 3 times higher than this of laminar flow since the velocity for the case of laminar flow is of order of 1 while the velocity for the case of compressible flow is of order of 0.0036 x 1000. Therefore, the averaged temperature $T_g(x)$ should be lower in the case of compressible flow because of the lower thickness of the thermal boundary layer. Moreover, this difference is always less than 5% and could be safely considered as insignificant. This underlines the negligible character of conduction in the gas phase because of the plateau observed in the temperature profile (very limited variation of T with x) due to the relatively high temperature regime that these processes occur. Generally speaking, the different flow regimes examined here are shown to produce thermal boundary layers of equivalent mean thickness under the restriction of constant mass flow rate.

Conclusions

The steady state heat transfer problem from the solid electrolyte to the absorbing cathode gas within a fuel cell was considered here. After a reduction of the dimensions of the original problem by using an averaging technique, the heat transfer equations accompanied by the appropriate boundary conditions are solved analytically for the gas phase. Three different flow regimes are considered: plug flow, fully developed incompressible laminar flow and compressible flow. Finally, the flow regime is found to play an insignificant role because the different flow regimes produce completely analogous temperature profiles having plateau almost everywhere along x. Thus, the consideration of the flow regime is of low importance in the process of design and implementation of fuel cell stacks.

References

[1] F.A. Coutelieris, S.Douvartzides and P. Tsiakaras: Chem. Eng. Sci. Vol. 60 (2005), p. 4423

[2] R.A. George and N.F. Bassette: J. Power Sources Vol. 71 (1998), p. 131

[3] J.H. Hirschenhofer, D.B. Stauffer and R.R. Engelman: *Fuel cells: a handbook (rev. 3)* (Morgantown: US DOE Office of Fossil Energy, USA 1998).

[4] R.H. Perry and D.W. Green: *Chemical Engineers Handbook* (7th Edition, McGraw-Hill, New York 1997).

[5] M. Tenenbaum and H. Pollard: *Ordinary Differential Equations* (4th Edition, Dover, New York 1985).

[6] C.L. Haynes and W.J. Wepfer: Int. J. Hydrogen En. Vol. 26 (2001), p. 369

Diffusion in Solids and Liquids V

doi:10.4028/www.scientific.net/DDF.297-301

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References

[1] F.A. Coutelieris, S.Douvartzides and P. Tsiakaras: Chem. Eng. Sci. Vol. 60 (2005), p. 4423
doi:10.1016/j.ces.2005.03.013

[2] R.A. George and N.F. Bassette: J. Power Sources Vol. 71 (1998), p. 131
doi:10.1016/S0378-7753(97)02735-3

[3] J.H. Hirschenhofer, D.B. Stauffer and R.R. Engelman: Fuel cells: a handbook (rev. 3) (Morgantown: US DOE Office of Fossil Energy, USA 1998).

[4] R.H. Perry and D.W. Green: Chemical Engineers Handbook (7th Edition, McGraw-Hill, New York 1997).

[5] M. Tenenbaum and H. Pollard: Ordinary Differential Equations (4th Edition, Dover, New York 1985).

[6] C.L. Haynes and W.J. Wepfer: Int. J. Hydrogen En. Vol. 26 (2001), p. 369
doi:10.1016/S0360-3199(00)00051-3